

Seesaw mechanism and structure of neutrino mass matrix

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Abstract

We consider the seesaw mechanism of neutrino mass generation in the light of our present knowledge of the neutrino masses and mixing. We analyse the seesaw mechanism constrained by the following assumptions: (1) minimal seesaw with no Higgs triplets, (2) hierarchical Dirac masses of neutrinos, (3) large lepton mixing primarily or solely due to the mixing in the right-handed neutrino sector, and (4) unrelated Dirac and Majorana sectors of neutrino masses. We show that large mixing governing the dominant channel of the atmospheric neutrino oscillations can be naturally obtained and point out that this constrained seesaw mechanism favours the normal mass hierarchy for the light neutrinos leading to a small V_{e3} entry of the lepton mixing matrix and a mass scale of the lightest right handed neutrino $M \simeq 10^{10} - 10^{11}$ GeV. Any of the three main neutrino oscillation solutions to the solar neutrino problem can be accommodated. The inverted mass hierarchy and quasi-degeneracy of neutrinos are disfavoured in our scheme.

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1 Introduction

The study of neutrino masses and lepton mixing may provide crucial clues towards the solution of the general problem of fermion masses and mixing. One of the striking features of lepton mixing is the fact that an explanation of the atmospheric neutrino anomaly [1] through neutrino oscillations requires a large mixing angle for $\nu_\mu \rightarrow \nu_\tau$ or $\nu_\mu \rightarrow \nu_{sterile}$. This is to be contrasted with the quark sector, where the mixing is small. Neutrinos have a distinguishing feature of being the only known fermions which are neutral with respect to all conserved charges, namely electric charge and colour. As a result, they can have Majorana masses, which in particular can arise through the seesaw mechanism [2].

The seesaw mechanism provides a very natural and attractive explanation of the smallness of the neutrino masses compared to the masses of the charged fermions of the same generation through the existence of heavy singlet neutrinos ν_R ¹. However, this mechanism does not fix completely the overall scale of the light neutrino masses since the mass scale of ν_R , though naturally large, is not precisely known. Moreover, the ratios of the light neutrino masses as well as the lepton mixing angles remain arbitrary: one can easily obtain any desired values of these parameters by properly choosing the neutrino Dirac mass matrix and Majorana mass matrix of singlet neutrinos. Therefore by itself, without any additional assumptions, this mechanism has limited predictive power. To gain more predictivity one has to invoke additional assumptions. In this letter, we study how phenomenologically viable neutrino masses and mixings can be generated within the framework of the seesaw mechanism, together with a reasonable set of assumptions, which can be summarized as follows:

(i) We work in the framework of three generation $SU(2)_L \times U(1)$ model, with the addition of three right-handed neutrino fields, which are singlets under $SU(2)_L \times U(1)$. No Higgs triplets are introduced and thus the effective mass matrix for the left-handed Majorana neutrinos is entirely generated by the seesaw mechanism, being given by

$$m_L = -m_D M_R^{-1} m_D^T, \quad (1)$$

where m_D denotes the neutrino Dirac mass matrix and M_R stands for the Majorana mass matrix of right-handed neutrinos.

(ii) We assume that the neutrino Dirac mass matrix m_D has a hierarchical eigenvalue structure, analogous to the one for the up-type quarks. This is a GUT-motivated assumption. However, for our arguments, the only important point is that the eigenvalues of m_D be hierarchical, their exact values do not play an essential rôle.

(iii) We assume that the charged lepton and neutrino Dirac mass matrices, m_l and m_D , are “aligned” in the sense that in the absence of the right-handed mass M_R , the leptonic mixing would be small, as it is in the quark sector. In other words, we assume that the

¹For recent studies of the seesaw mechanism see, e.g., [3, 4, 5].

left-handed rotations that diagonalize m_l and m_D are the same or nearly the same. Again, this assumption is motivated by GUTs. We therefore consider that the large lepton mixing results from the fact that neutrinos acquire their mass through the seesaw mechanism.

(iv) We assume that the Dirac and Majorana neutrino mass matrices are unrelated. The exact meaning of this assumption will be explained in the next section.

We shall investigate whether the seesaw mechanism, constrained by our set of assumptions, can lead to a phenomenologically viable neutrino mass matrix and, if so, whether it can help us to understand some of the salient features of the leptonic mixing. In particular, it would be interesting to understand why the mixing angle θ_{23} responsible for the atmospheric $\nu_\mu \leftrightarrow \nu_\tau$ oscillations is large, while the mixing angle θ_{13} which governs the subdominant $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations of atmospheric neutrinos and long baseline $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations is small. Another interesting question is whether this constrained seesaw mechanism can help us to discriminate among possible neutrino mass hierarchies – normal hierarchy, inverted hierarchy and quasi-degeneracy. Furthermore, it would be useful if the seesaw mechanism could provide some guidance as to the possible solutions to the solar neutrino problem – large mixing angle MSW (LMA), small mixing angle MSW (SMA) and vacuum oscillations (VO) solutions. We shall address these issues within the constrained seesaw mechanism described above.

2 General framework

As previously mentioned, we work in the context of the standard three generations $SU(2)_L \times U(1)$ model, where the only additional fields are the three right-handed neutrinos ν_{iR} ². The most general charged lepton and neutrino mass terms can be written as

$$\mathcal{L}_{mass} = (m_l)_{ij} \bar{l}_{iL} l_{jR} + (m_D)_{ij} \bar{\nu}_{iL} \nu_{jR} + \frac{1}{2} (M_R)_{ij} \nu_{iR}^T C \nu_{jR} + h.c., \quad (2)$$

where m_l and m_D stand for the charged lepton and neutrino Dirac mass matrices arising from Yukawa coupling with the Higgs doublet, while M_R denotes the Majorana mass matrix of right-handed neutrinos. Since the right-handed Majorana mass terms are $SU(2)_L \times U(1)$ invariant, M_R is naturally large, not being protected by the low energy gauge symmetry. The matrices m_l and m_D are in general arbitrary complex matrices, while M_R is a symmetric complex matrix. Without loss of generality we may choose a weak basis (WB) where the charged lepton mass matrix is diagonal, with real positive eigenvalues. The lepton mass matrices can be written as

$$m_l \equiv d_l = \text{diag}(m_e, m_\mu, m_\tau),$$

²Only two of the three known experimental indications of nonzero neutrino mass (solar neutrino problem [6], atmospheric neutrino data [1] and the accelerator LSND results [7]) can be explained through neutrino oscillations with just three light neutrino species. As the LSND result is the only one that has not yet been independently confirmed, we choose not to consider it here.

$$\begin{aligned} m_D &= V_L d_\nu V_R^\dagger, \\ M_R &= U_R D U_R^T, \end{aligned} \quad (3)$$

where d_l , d_ν and D are diagonal, real positive matrices while V_L , V_R and U_R are unitary matrices. In the absence of M_R , the leptonic mixing matrix V entering in the charged-current weak interactions would be given by $V = V_L$. Our assumption (iii) that m_l and m_D are “aligned” in their left-handed rotations means that V_L is assumed to be close to the unit matrix, thus implying that in the absence of M_R leptonic mixing would be small, in analogy with the quark sector.

The mass terms in Eq. (3) are written in a WB, therefore the gauge currents are still diagonal. It should be emphasized that one has a large freedom to make WB transformations which leave the gauge currents diagonal but alter the mass terms. One can use this freedom to choose, e.g., a WB basis where both m_l and M_R are diagonal. However, for our arguments, it will be more convenient to choose a different ν_R basis, to be specified below.

The effective mass matrix of the light left-handed neutrinos resulting from the seesaw mechanism can then be written as

$$m_L = -V_L d_\nu W_R D^{-1} W_R^T d_\nu V_L^T, \quad (4)$$

where $W_R = V_R^\dagger U_R^*$. The physical leptonic mixing among the light neutrinos which enters in the probabilities of neutrino oscillations is given by the matrix V that diagonalizes m_L :

$$V^T m_L V = \text{diag}(m_1, m_2, m_3). \quad (5)$$

Here m_i ($i = 1, 2, 3$) are the masses of the light neutrinos. We shall disregard possible CP violation effects in the leptonic sector and assume the neutrino mass matrix to be real. Its eigenvalues m_i can be of either sign, depending on the relative CP parities of neutrinos. The physical neutrino masses are $|m_i|$.

One of the challenges is how to obtain large mixing in V without resorting to fine tuning. We shall show that this is possible in the framework of the seesaw mechanism, together with the assumptions listed in sec. 1.

Following our assumption (iii), we shall consider that $V_L \approx 1$ in the WB where m_l is diagonal. One can then write

$$m_L = -d_\nu (M'_R)^{-1} d_\nu, \quad (6)$$

where

$$(M'_R)^{-1} = W_R D^{-1} W_R^T, \quad (7)$$

thus fixing the ν_R basis. It is useful to write the explicit form of m_L as

$$m_L = - \begin{pmatrix} m_u^2 M_{11}^{-1} & m_u m_c M_{12}^{-1} & m_u m_t M_{13}^{-1} \\ m_u m_c M_{12}^{-1} & m_c^2 M_{22}^{-1} & m_c m_t M_{23}^{-1} \\ m_u m_t M_{13}^{-1} & m_c m_t M_{23}^{-1} & m_t^2 M_{33}^{-1} \end{pmatrix}, \quad (8)$$

where $M_{ij}^{-1} \equiv (M'_R)_{ij}^{-1}$, and m_u, m_c and m_t denote the eigenvalues of m_D . For our numerical estimates we shall take them to be equal to the masses of the corresponding up-type quarks, but for our general arguments their precise values are unimportant.

We shall adopt the parametrization of the leptonic mixing matrix V which coincides with the standard parametrization of the quark mixing matrix [8] and identify the mixing angle responsible for the dominant channel of the atmospheric neutrino oscillations with θ_{23} , the one governing the solar neutrino oscillations with θ_{12} and the mixing angle which governs the subdominant $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations of atmospheric neutrinos and long baseline $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations with θ_{13} . Assuming that $m_1, m_2 \ll m_3$ and $\theta_{23} \simeq 45^\circ$ (which is the best fit value of the Super-Kamiokande data [9]) and taking into account that the CHOOZ experiment indicates that $\theta_{13} \ll 1$ [10], it can be shown that m_L must have the approximate form [11]

$$m_L = m_0 \begin{pmatrix} \kappa & \varepsilon & \varepsilon' \\ \varepsilon & 1 + \delta - \delta' & 1 - \delta \\ \varepsilon' & 1 - \delta & 1 + \delta + \delta' \end{pmatrix}, \quad (9)$$

where $\kappa, \varepsilon, \varepsilon', \delta$ and δ' are small dimensionless parameters. Comparing Eqs. (8) and (9), one concludes that the following relations should hold, in leading order:

$$m_c^2 M_{22}^{-1} = m_t^2 M_{33}^{-1} = m_c m_t M_{23}^{-1}. \quad (10)$$

These relations seem to indicate that in order to obtain the form of Eq. (9), strong correlations are required between the entries of d_ν and those of $(M'_R)^{-1}$, in apparent contradiction with our assumption (iv). However, it can be readily seen that in fact there is no conflict. Obviously, the form of $(M'_R)^{-1}$ depends on the ν_R basis one chooses. In the definition of $(M'_R)^{-1}$ given by Eq. (7), we have included part of the right-handed rotation arising from the diagonalization of m_D , namely V_R enters in W_R defined as $W_R = V_R^\dagger U_R^*$. Therefore $(M'_R)^{-1}$ contains information about the Dirac mass sector, and Eq. (10) is not necessarily in conflict with our assumption (iv). This assumption has to be formulated in terms of weak-basis invariants. What should be required is that the ratios of the *eigenvalues* of $(M'_R)^{-1}$ should not be related to the ratios of the *eigenvalues* of m_D .

In order to see how the phenomenologically favoured form of m_L can be achieved without contrived fine tuning between the parameters of the Dirac and Majorana sectors, let us first consider the two-dimensional sector of m_L in the 2-3 subspace, which is responsible for a large θ_{23} . We shall write the diagonalized Dirac mass matrix d_ν using the dimensionless parameters p and q :

$$d_\nu = m_t \text{diag}(p^2 q, p, 1), \quad p = m_c/m_t \sim 10^{-2}, \quad q = m_u m_t/m_c^2 \sim 0.4. \quad (11)$$

It follows from Eq. (6) that the 2-3 sector of $(M'_R)^{-1}$, in order to lead to the 2-3 structure of Eq. (9) (with all elements approximately equal to unity up to a common factor), should have the following form:

$$M_R^{-1} \propto \begin{pmatrix} 1 & p \\ p & p^2 \end{pmatrix}. \quad (12)$$

The eigenvalues of the matrix in Eq. (12) are 0 and $1 + p^2$, and thus by choosing the pre-factor to be $const/(1 + p^2)$ one arrives at the matrix M_R^{-1} of the desired form with p - and q -independent eigenvalues. The question is now whether it is possible to find a 3×3 matrix whose 2-3 sector corresponds to Eq. (12) while its eigenvalues (or invariants) are independent of p and q . This turns out to be possible, and the simplest form of $(M'_R)^{-1}$ fulfilling this requirement is

$$(M'_R)^{-1} \propto \frac{1}{1 + p^2} \begin{pmatrix} (1 + p^2) \gamma & \sqrt{1 + p^2} (\beta - \alpha p) & \sqrt{1 + p^2} (\alpha + \beta p) \\ \sqrt{1 + p^2} (\beta - \alpha p) & 1 & p \\ \sqrt{1 + p^2} (\alpha + \beta p) & p & p^2 \end{pmatrix}, \quad (13)$$

where the dimensionless parameters α , β and γ do not depend on p and q . It is straightforward to check that the eigenvalues of the matrix in Eq. (13) are p - and q -independent. The easiest way to do that is by noting that $(M'_R)^{-1}$ can be written as

$$(M'_R)^{-1} = S_R^T (M_R^0)^{-1} S_R \quad (14)$$

with

$$S_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & -s_\phi & c_\phi \end{pmatrix}, \quad (M_R^0)^{-1} = \frac{1}{2M} \begin{pmatrix} \gamma & \beta & \alpha \\ \beta & 1 & 0 \\ \alpha & 0 & 0 \end{pmatrix}, \quad (15)$$

and

$$c_\phi = \cos \phi, \quad s_\phi = \sin \phi, \quad \phi = \arctan p. \quad (16)$$

From Eqs. (6), (14), (15) and (16) one obtains

$$m_L = \frac{m_t^2}{2M} \frac{p^2}{1 + p^2} \begin{pmatrix} q'^2 p^2 \gamma & q' p (\beta - \alpha p) & q' (\alpha + \beta p) \\ q' p (\beta - \alpha p) & 1 & 1 \\ q' (\alpha + \beta p) & 1 & 1 \end{pmatrix}, \quad (17)$$

where

$$q' \equiv q \sqrt{1 + p^2} \simeq q. \quad (18)$$

It is worth emphasizing that we have obtained m_L of the desired form, while abiding by our assumptions. Comparison of Eqs. (9) and (17) leads to the following identification for the parameters of the phenomenological mass matrix of light neutrinos:

$$\kappa = q'^2 p^2 \gamma, \quad \varepsilon = q' p (\beta - \alpha p), \quad \varepsilon' = q' (\alpha + \beta p), \quad \delta = \delta' = 0. \quad (19)$$

The largest eigenvalue of the matrix m_L in Eq. (17), i.e. the mass of the heaviest of the three light neutrinos is

$$m_3 \simeq \frac{m_t^2}{M} \frac{p^2}{1 + p^2} \simeq \frac{m_c^2}{M}. \quad (20)$$

It scales as m_c^2 rather than as usually expected m_t^2 . It has to be identified with $\Delta m_{atm}^2 \simeq (2 - 6) \times 10^{-3} \text{ eV}^2$, which gives

$$M \simeq (10^{10} - 10^{11}) \text{ GeV}, \quad (21)$$

i.e. an intermediate mass scale rather than the GUT scale.

It has been shown in [11] that the MSW effect [12] can only occur for neutrinos, and in particular the LMA and SMA solutions of the solar neutrino problem are only possible, if the parameters of the mass matrix m_L in Eq. (9) satisfy

$$|4\delta - \delta'^2| > |2\kappa - (\varepsilon^2 + \varepsilon'^2)|. \quad (22)$$

Since $\delta = \delta' = 0$ in Eq. (17), it is clear that the only solution to the solar neutrino problem that is not automatically ruled out in the case under consideration is the VO solution.

It is interesting to ask whether our scheme can be modified so that nonzero values for δ and δ' be obtained. One simple possibility would be to assume an incomplete alignment between the mass matrix of charged leptons and the Dirac mass matrix of neutrinos: $V_L \approx 1$ instead of $V_L = 1$. Then the effective mass matrix of light neutrinos m_L would be obtained from Eq. (17) by the additional rotation by V_L . Taking for simplicity this rotation to be in the 2-3 subspace, one can readily make sure that it indeed yields nonzero δ and δ' . However, in this case they are related by $\delta = \delta'^2/4$. Therefore the left-hand side (l.h.s.) of (22) vanishes, i.e. this inequality is not satisfied and the MSW solutions of the solar neutrino problem are still not possible. The fact that an additional rotation in the 2-3 subspace does not change the l.h.s. of (22) becomes obvious by noticing that $4\delta - \delta'^2$ coincides with the determinant of the 2×2 submatrix of Eq. (9) in the 2-3 subspace. Thus, to accommodate the LMA or SMA solutions of the solar neutrino problem through the V_L rotation one should consider a matrix V_L of a more general form.

There is, however, another way to achieve the same goal. One can arrive at $\delta, \delta' \neq 0$ even with $V_L = 1$ if one considers the following simple modification of the the matrix $(M_R^0)^{-1}$ in Eq. (15):

$$(M_R^0)^{-1} = \frac{1}{2M} \begin{pmatrix} \gamma & \beta & \alpha \\ \beta & 1 & 0 \\ \alpha & 0 & r \end{pmatrix}, \quad (23)$$

i.e. the 33-element of the matrix now is nonzero. The requirement $|\delta|, |\delta'| \ll 1$ translates into $|r| \ll p^2$. This yields the following effective mass matrix for the light neutrinos:

$$m_L \simeq \frac{m_t^2}{2M} \frac{p^2}{1+p^2} \begin{pmatrix} q'^2 p^2 \gamma & q'p(\beta - \alpha p) & q'(\alpha + \beta p) \\ q'p(\beta - \alpha p) & 1 - r/4p^2 & 1 - r/4p^2 \\ q'(\alpha + \beta p) & 1 - r/4p^2 & 1 + 3r/4p^2 \end{pmatrix}, \quad (24)$$

This means that now

$$\delta \simeq r/4p^2, \quad \delta' \simeq r/2p^2, \quad (25)$$

and so the l.h.s. of (22) is nonzero, i.e. SMA and LMA solutions of the solar neutrino problem are possible. The parameters κ , ε and ε' in this case are the same as in the case $r = 0$, i.e. are given by Eq. (19)³.

³The particular case of the neutrino mass matrix of the form (24) with $\beta = \gamma = r = 0$ (which allows only the VO solution of the solar neutrino problem) was obtained in [4].

3 Numerical examples

We shall now give some illustrative examples of the values of the parameters for which all three types of the solutions – SMA, LMA and VO – are possible. We concentrate here on the case of normal mass hierarchy ($m_1, m_2 \ll m_3$); the cases of inverted mass hierarchy and quasi-degeneracy will be discussed in sec. 4.

The neutrino mass squared differences which enter in the probabilities of the solar and atmospheric neutrino oscillations are related to the eigenvalues of the effective mass matrix of light neutrinos m_L via $\Delta m_\odot^2 \simeq \Delta m_{21}^2$, $\Delta m_{atm}^2 \simeq \Delta m_{31}^2 \simeq m_3^2$. Consider first the following choice of the parameters of the matrix $(M_R^0)^{-1}$ in Eq. (23): $\alpha = 1.1 \cdot 10^{-2}$, $\beta \sim \alpha$, $\gamma \lesssim \beta^2$, $r = 8 \cdot 10^{-6}$. This gives the following values of the parameters of the mass matrix of light neutrinos m_L in (9): $\kappa \lesssim 10^{-9}$, $\varepsilon \simeq 3 \cdot 10^{-5}$, $\varepsilon' \simeq 4.5 \cdot 10^{-3}$, $\delta \simeq 0.04$, $\delta' \simeq 0.08$. Diagonalization of m_L can then be easily performed either by making use of the approximate analytic expressions derived in [11] or numerically. It gives the following values of the masses and mixings of light neutrinos:

$$m_1 \simeq -3.6 \cdot 10^{-6} \text{eV}, \quad m_2 \simeq 2.3 \cdot 10^{-3} \text{eV}, \quad m_3 \simeq 2m_0 \simeq 0.06 \text{eV},$$

$$\sin^2 2\theta_{12} \simeq 6.2 \cdot 10^{-3}, \quad \sin \theta_{13} \simeq 1.7 \cdot 10^{-3}, \quad (26)$$

where we have taken $m_0 \simeq \sqrt{\Delta m_{atm}^2}/2 \simeq 0.03 \text{eV}$. In all the cases we consider, the value of the mixing angle θ_{23} is very close to 45° by construction of our mass matrix m_L . From Eq. (26) one finds $\Delta m_\odot^2 \simeq 5.3 \cdot 10^{-6} \text{eV}^2$, i.e. this choice of the parameters leads to the SMA solution of the solar neutrino problem⁴. The corresponding mass eigenvalues of heavy Majorana neutrinos are $M_1 \simeq 6 \cdot 10^{10} \text{GeV}$, $M_2 \simeq -M_3 \simeq 5.5 \cdot 10^{11} \text{GeV}$.

Let us now choose $\alpha = -0.75$, $\beta \sim \alpha$, $\gamma \lesssim \beta^2$, $r = 1.5 \cdot 10^{-5}$. This yields $\kappa \lesssim 4 \cdot 10^{-6}$, $\varepsilon \simeq 2 \cdot 10^{-3}$, $\varepsilon' \simeq 0.3$, $\delta \simeq 0.073$, $\delta' \simeq 0.146$. Diagonalization of m_L then gives

$$m_1 \simeq -4.62 \cdot 10^{-3} \text{eV}, \quad m_2 \simeq 7.86 \cdot 10^{-3} \text{eV}, \quad m_3 \simeq 2m_0 \simeq 0.06 \text{eV},$$

$$\sin^2 2\theta_{12} \simeq 0.83, \quad \sin \theta_{13} \simeq 0.11, \quad (27)$$

with $\Delta m_\odot^2 \simeq 4 \cdot 10^{-5} \text{eV}^2$, i.e. this choice of the parameters leads to the LMA solution of the solar neutrino problem. The corresponding mass eigenvalues of heavy Majorana neutrinos are $M_1 \simeq 4.6 \cdot 10^{10} \text{GeV}$, $M_2 \simeq -7.4 \cdot 10^{10} \text{GeV}$, $M_3 \simeq 1.1 \cdot 10^{11} \text{GeV}$.

Finally, let us choose $\alpha = 1.13 \cdot 10^{-3}$, $\beta \lesssim \alpha$, $\gamma \lesssim \beta^2$, $r = 2 \cdot 10^{-8}$. This yields $\kappa \lesssim 10^{-11}$, $\varepsilon \lesssim 3 \cdot 10^{-6}$, $\varepsilon' \simeq 4.5 \cdot 10^{-4}$, $\delta \simeq 10^{-4}$, $\delta' \simeq 2 \cdot 10^{-4}$. Diagonalization of m_L gives

$$m_1 \simeq -6.95 \cdot 10^{-6} \text{eV}, \quad m_2 \simeq 1.29 \cdot 10^{-5} \text{eV}, \quad m_3 \simeq 2m_0 \simeq 0.06 \text{eV},$$

$$\sin^2 2\theta_{12} \simeq 0.91, \quad \sin \theta_{13} \simeq 1.6 \cdot 10^{-4}, \quad (28)$$

⁴For recent analyses of the solar neutrino data and allowed ranges of the parameters see [13, 14, 15].

with $\Delta m_\odot^2 \simeq 1.2 \cdot 10^{-10} \text{ eV}^2$, i.e. this choice of the parameters leads to the VO solution of the solar neutrino problem. The corresponding mass eigenvalues of heavy Majorana neutrinos are $M_1 \simeq 6 \cdot 10^{10} \text{ GeV}$, $M_2 \simeq -M_3 \simeq 5.3 \cdot 10^{13} \text{ GeV}$. Alternatively, one could choose $\alpha = 1.75 \cdot 10^{-2}$, $\beta \lesssim \alpha$, $\gamma \lesssim \beta^2$, $r = 0$. This gives $\kappa \lesssim 2.5 \cdot 10^{-9}$, $\varepsilon \lesssim 5 \cdot 10^{-5}$, $\varepsilon' \simeq 7 \cdot 10^{-3}$, $\delta = \delta' = 0$. One then obtains $\Delta m_\odot^2 \simeq 1.1 \cdot 10^{-10} \text{ eV}^2$, $\sin \theta_{13} \simeq 2.5 \cdot 10^{-3}$ and $\theta_{12} \simeq \theta_{23} \simeq 45^\circ$, i.e. this choice of the parameters leads to the VO solution of the solar neutrino problem with bi-maximal neutrino mixing [16, 4]. The mass eigenvalues of heavy Majorana neutrinos in this case are $M_1 \simeq 6 \cdot 10^{10} \text{ GeV}$, $M_2 \simeq -M_3 \simeq 3.4 \cdot 10^{12} \text{ GeV}$.

The examples given here demonstrate that with the inverse mass matrix of right-handed neutrinos $(M'_R)^{-1}$ of the form (23), depending on the values of its parameters, all three main neutrino oscillations solutions to the solar neutrino problem – SMA, LMA and VO – can be realized in the framework of the constrained seesaw mechanism.

4 Inverted mass hierarchy and quasi-degeneracy

We shall now briefly discuss the other possible neutrino mass hierarchies – the inverted mass hierarchy $|m_3| \ll |m_1| \simeq |m_2|$ and the quasi-degenerate case with $|m_1| \simeq |m_2| \simeq |m_3|$. For the case of the inverted mass hierarchy there are three main zeroth order textures corresponding to the limit $|m_1| = |m_2|$, $\theta_{13} = 0$, $\theta_{23} = 45^\circ$ (see, e.g., [17]):

$$m_L \propto \begin{pmatrix} \pm 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}; \quad \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (29)$$

The first two textures differ only by the sign of the 11-term. One can then invert Eq. (6) to find the inverse mass matrices of right-handed neutrinos $(M'_R)^{-1}$ which lead to these textures. To be consistent with our assumption (iv), these matrices must satisfy the following requirement: it should be possible to obtain each of them by a p - and q -dependent rotation of a p - and q -independent matrix. In other words, their eigenvalues Λ_i ($i = 1, 2, 3$) must be p - and q -independent.

Since all mass matrices in Eq. (29) have one zero eigenvalue, so do the corresponding matrices $(M'_R)^{-1}$, and their determinants vanish. Therefore it is sufficient to check if their traces $(\Lambda_1 + \Lambda_2 + \Lambda_3)$ and second invariants $(\Lambda_1\Lambda_2 + \Lambda_1\Lambda_3 + \Lambda_2\Lambda_3)$ can be made p - and q -independent by a proper choice of the pre-factors in Eq. (29). One can readily make sure that for the first two textures (first matrix in Eq. (29) with both signs of the 11 element) this is impossible: if the trace of the corresponding matrix $(M'_R)^{-1}$ is made p - and q -independent, then the second invariant depends on p and q , and vice versa. Therefore these zeroth-order textures do not satisfy our conditions (i) – (iv). Perturbing these textures by adding small terms to each their element will not change this conclusion.

The situation is different for the second matrix in Eq. (29). The corresponding matrix $(M'_R)^{-1}$ has zero determinant and trace, and the second invariant can be always made p -

and q -independent by a proper choice of the pre-factor. However, this texture is ruled out on different grounds. It describes the bi-maximal mixing with the inverted hierarchy, which means $\theta_{12} = 45^\circ$. In this case only the VO solution of the solar neutrino problem is possible [13, 14]. This implies $\Delta m_{21}^2 = \Delta m_\odot^2 \sim 10^{-10} \text{ eV}^2$. Small nonvanishing values of Δm_{21}^2 are achieved when zeros in the texture matrix are filled in with small terms, and for the VO solution these small terms should be $\lesssim 10^{-8}$. The diagonalization of the corresponding matrix $(M'_R)^{-1}$ then gives the following values of the masses of the heavy singlet neutrinos $M_i = \Lambda_i^{-1}$: two singlet neutrinos are almost degenerate with $M_1 \simeq -M_2 \sim 10^8 \text{ GeV}$; the third mass eigenvalue turns out to be well above the Planck scale: $M_3 \sim 10^{22} \text{ GeV}$, clearly not a physical value. Thus, this case of the inverted mass hierarchy is ruled out as well. It is interesting to note that in our argument we have not used the condition (iv), i.e. the mass matrices obtained from the last texture in Eq. (29) are excluded on the basis of our assumptions (i)-(iii) only. The mass matrices leading to the quasi-degenerate neutrino mass spectrum can be considered and ruled out using arguments analogous to those applied to the cases of the first two textures in Eq. (29).

5 Discussion

We have shown that the seesaw mechanism, supplemented by the set of assumptions listed in the Introduction, leads to phenomenologically viable mass matrices of light active neutrinos. The mixing angle θ_{23} responsible for the dominant channel of the atmospheric neutrino oscillations can be naturally large without any fine tuning. The fact that the seesaw mechanism can lead to large lepton mixing even if the mixing in both the Dirac and Majorana sectors is small has been known for some time [5, 18, 19]. Our approach gives a simple explanation to this fact: if the eigenvalues of the Dirac mass matrix of neutrinos m_D are hierarchical, and the entries M_{ij}^{-1} of the inverse Majorana mass matrix $(M'_R)^{-1}$ have the hierarchy $M_{22}^{-1} \gg M_{23}^{-1} \gg M_{33}^{-1}$ (which implies small mixing in the 2-3 sector of right-handed Majorana neutrinos), then the multiplication of $(M'_R)^{-1}$ by m_D from the left and from the right suppresses the 22 element of the resulting effective mass matrix m_L to a larger extent than it suppresses the 23 element, which in turn is more suppressed than the 33 element. This can lead to all the elements of the 2-3 sector of the resulting matrix m_L being of the same order, yielding a large mixing angle θ_{23} .

Although the constrained seesaw mechanism allows to obtain a large mixing angle θ_{23} in a very natural way, it does not explain why θ_{23} is large: the largeness of this mixing angle is merely related to the choice of the inverse mass matrix of heavy singlet neutrinos, Eq. (15) or (23). However, once this choice has been made, the smallness of the mixing angle θ_{13} which determines the element V_{e3} of the lepton mixing matrix can be readily understood. For the case of the normal mass hierarchy $m_1, m_2 \ll m_3$ the value of θ_{13} can be expressed in terms of the entries of the effective mass matrix m_L in Eq. (9) as $\sin \theta_{13} \simeq (\varepsilon + \varepsilon')/2\sqrt{2}$ [11]. From Eq. (19) one then finds $\sin \theta_{13} \simeq q(\alpha + 2\beta p)/2\sqrt{2} \simeq q\alpha/2\sqrt{2} \simeq 0.14\alpha$, assuming

$\beta \ll \alpha p^{-1} \sim 100\alpha$. Since all the solutions of the solar neutrino problem require $|\alpha| < 1$ in order to have small enough Δm_{\odot}^2 (see sec. 3), the smallness of θ_{13} follows.

We have shown that all three main neutrino oscillations solutions to the solar neutrino problem – small mixing angle MSW, large mixing angle MSW and vacuum oscillations – are possible within the constrained seesaw. The mechanism does not favour any of these solutions over the others.

The seesaw mechanism we have studied naturally leads to the normal neutrino mass hierarchy while disfavouring the inverted mass hierarchy and quasi-degenerate neutrinos. For LMA and SMA solutions of the solar neutrino problem, the masses of the heavy singlet neutrinos are of the order $10^{10} - 10^{11}$ GeV. For the VO solution, the lightest of the singlet neutrinos has the mass of the same order of magnitude, whereas the masses of the other two are $\sim 10^{12} - 10^{13}$ GeV.

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